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A Fault Tolerance Method for Control Systems with Full or Partial Fault Decoupling

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Abstract—This paper considers technical systems described by nonlinear dynamic models. The fault tolerance property of such systems is ensured by introducing feedback with full or partial fault decoupling. The solution is based on separating a subsystem insensitive or minimally sensitive to faults and its subsequent analysis. For this purpose, a logical-dynamic approach is used, which operates only linear algebra methods. An illustrative practical example is provided.

Keywords: nonlinear systems, faults, isolation, feedback, singular value decomposition

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1. INTRODUCTION

Modern technical systems (robots, control systems) are subjected to various faults in their elements. Redundancy is one way to eliminate the effect of such faults [1]; however, it requires excessive resources and is not always implementable in practice. The use of fault diagnosis methods is a more promising approach to improving the reliability, safety, and efficiency of such systems. In real time, these methods have to detect emerging faults and determine the values of the changed system parameters and errors in the readings of their sensors. After that, all identified changes with undesirable consequences are promptly parried.

Various fault diagnosis methods were thoroughly described in [2], including the basic terminology in this area. According to [2], a fault is understood as an unacceptable deviation of at least one of the characteristic properties or variables of a system from its standard (nominal) behavior. In this paper, such a deviation is represented by an unknown bounded time-varying function $d(t)$ added to certain components of the system state vector depending on the fault location.

As is known [3], adaptive systems designed to parry the consequences of faults and changes in the parameters of control objects can be divided into two large groups: systems with self-adjusting structure (self-organizing systems) and systems with self-tuning parameters (self-tuning systems). In the former case, certain structural changes are made to the system being diagnosed, i.e., it is reconfigured to remove failed elements and use redundant ones. In the latter case, depending on the changes in the parameters of the control object, emerging faults, or external influences, only certain parameters of the used controller are tuned according to some algorithm embedded in the self-tuning device. The system with faults and changed parameters should continue functioning, preserving its most important characteristics within the admissible limits.

Each of the above approaches has peculiarities, which somewhat restrict the scope of their practical application. In particular, the possibility of involving redundant elements is limited by the maximal design-achievable and operational (mass and size, energy, etc.) characteristics of specific robots.

Examples of implementing such an approach were described in [4, 5]. The cited authors solved the problem of fault-tolerant control of underwater robots in case of failure of one thruster (the first work mentioned) and in case of faulty electric actuators installed in the manipulator joints (the second one). In both cases, it was proposed to disconnect the faulty actuator and then distribute its control actions between the others with additional connection of the redundant ones. The disadvantage of such systems is the need for extra actuators in robots, which complicates the design and appreciably increases the cost of robots. In addition, the feasibility of using redundancy must be justified by additional calculations of reliability indicators. As a rule, redundancy elements have the same reliability as the replaced ones; as a result, the possibility of increasing the reliability of robots through redundancy is significantly limited. Fault adaptation methods based on self-tuning allow avoiding additional hardware costs, but their use admits degradation of some (usually minor) performance indicators of robots, possibly affecting the tactical and technical characteristics of robots and, in some cases, even requiring correction of the mission.

Fault-tolerant self-tuning systems with a reference model are known; their design principles were presented in [6, 7]. The main peculiarity of this class of systems is the availability of an explicit technical device (model) with given dynamic properties. In this case, the dynamics of the entire system are reduced to the desired dynamics of the model. Such adaptation systems to faults and variable parameters have found application in both ground and underwater robotics [8–10], providing high-quality control of robots with rather simple means without identifying the parameter deviations caused by faults or other external factors during their operation. As the main drawback of such systems, we note the presence of high-frequency oscillations in the self-tuning loop, which in some cases may significantly reduce the quality of adaptation to emerging faults and variable parameters. In addition, during the operation of such systems, the deviations of parameters from their nominal values are not determined; therefore, in the case of critical faults (e.g., short-circuit in some winding turns of the anchor chain of electric motors, the appearance of significant external torques on motor shafts), the robots will not be promptly stopped, and their further breakdown will not be prevented. The systems under consideration also neglect errors in the readings of robot sensors.

Optimal and robust principles of adaptive systems design are often used in engineering to compensate for the consequences of emerging faults and parameter deviations from nominal values [11–13]. The advantage of such systems is a sufficiently high level of robustness to the uncertain parameters of robots, but they are built based on a linearized model, which restricts their application to fault-tolerant control of the spatial motion of complex dynamic objects.

Currently, variable-structure systems operating in sliding mode are a common type of robust control systems. Examples of their use for fault-tolerant robot control were described in [14–17]. Control systems with adaptation to emerging faults and parameter deviations based on variable-structure systems have several considerable benefits compared to other types of fault-tolerant systems. Despite this fact, they also suffer from the disadvantage that, in order to ensure the performance of a variable-structure system within the entire range of changes in robot parameters, such systems are designed in the worst case (when these parameters correspond to the lowest system performance). As a result, even in the absence of faults, additional control signals are generated, which will increase their amplitude and energy consumption and, consequently, reduce the autonomous operation time. That is, fault-tolerant control systems of this class have a deliberately underestimated performance.

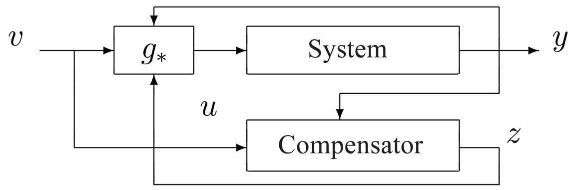

Fig. 1. The implementation scheme of the proposed solution.

The approaches and methods discussed above are illustrated mainly by examples of robots, but their peculiarities also apply to many modern technical systems.

A certain alternative to the considered methods is approaches based on full or partial fault decoupling: a fault is detected, but the values of the changed system parameters are not determined, and the control action on the system is corrected by using a specially built compensator and a new control. As a result, the system will execute its main operations with the previous or admissibly reduced quality. By assumption, the execution of these operations depends not on all components of the system state vector but only on some part of them, defined by a known function, and these components have to be fully or partially decoupled from possible system faults.

Figure 1 shows the implementation scheme of the proposed solution, where $u(t)$ and $y(t)$ are the control vector and output of the system, respectively, $z(t)$ is the state vector of the compensator, $v(t)$ is the new control, and g_* is a function defined below. The control $u(t)$ was constructed to execute certain operations by the system, and the new control $v(t)$ must be constructed to execute the same operations by the system with the compensator, with the same or admissibly reduced quality.

This approach has certain limitations: figuratively speaking, it can be implemented if there exists a control signal between the fault location and the system variables that need to be decoupled from this fault; the control signal is used for fault decoupling.

For systems described by nonlinear difference equations, such an approach was implemented in [18, 19] based on full decoupling using a rather complex mathematical apparatus of function algebra. In distinction, this paper considers systems given by nonlinear differential equations subject to faults. For such a system, it is required to find a description of the compensator and a function g_* to decouple from faults, fully or partially, given components of the system state vector.

The problem of determining the new control $v(t)$ is not considered below since this control depends on the tasks solved by the system and can be determined when specifying these tasks. After the compensator is built, the new control can be determined by known methods [20]; the compensator depends on given components of the system state vector and the fault location and is independent of the tasks solved by the system.

Note that for affine systems, such a problem was solved in [21] based on full decoupling by rather complicated methods of differential geometry. The novelty of this paper is that the systems under consideration may contain unsmooth nonlinearities; the problem is solved using the logical-dynamic approach [22], which allows analyzing nonlinear systems by linear algebra methods under definite restrictions on the class of solutions. Moreover, partial fault decoupling is studied in addition to full decoupling.

The remainder of this paper is organized as follows. Section 2 presents the main models: descriptions of the given nonlinear system and its submodel used to build the compensator. In Section 3, a fault-insensitive submodel is constructed; in Section 4, a submodel minimally sensitive to faults. Section 5 is devoted to the compensator design. An illustrative example is provided in Section 6, and Section 7 concludes the paper.

2. MAIN MODELS

Consider systems described by the nonlinear model

$$\begin{aligned} \dot{x}(t) &= Fx(t) + Gu(t) + C\Psi(x(t), u(t)) + Dd(t), \\ y(t) &= Hx(t), \end{aligned} \tag{2.1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^l$ are the state vector, control input, and output, respectively; F and G are known constant matrices that describe linear dynamics; H, C, and D are known constant matrices; $d(t)$ is a scalar function that describes faults (if there are no faults, $d(t) = 0$; when faults occur, $d(t)$ becomes an unknown bounded time-varying function); $\Psi(x, u)$ is the nonlinear part represented as

$$\Psi(x,u) = \begin{pmatrix} \varphi_1(A_1 x, u) \\ \dots \\ \varphi_q(A_q x, u) \end{pmatrix},$$

where $A_1, \dots, A_q$ are known constant row matrices, and $\varphi_1, \dots, \varphi_q$ are arbitrary nonlinear functions.

Remark 1. If the system may have several faults, then (generally speaking) it is necessary to build a bank of several compensators for fault decoupling. The method under consideration cannot be applied to decouple from sensor faults; if the value of such a fault is unknown, it is necessary to exclude the readings of the corresponding sensor from the control system or use a virtual sensor instead [23].

Note that the nonlinear system (2.1) can be obtained from the general nonlinear system

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t), d(t)), \\ y(t) &= h(x(t)) \end{aligned} \tag{2.2}$$

by several transformations [22].

By assumption, faults in the system change the value of some system parameter. As a result, $d(t)$ represents the product of this change by some component of the vector $x(t)$ or $u(t)$ and is an unknown bounded time-varying function; the matrix D indicates the fault location. The fault can be detected and isolated by known fault diagnosis methods (e.g., see [2]), but the function $d(t)$ still remains unknown.

By another assumption, the function of the components of the system state vector $x(t)$ for full or partial fault decoupling is given by a known matrix H_0 defining the variable $y_0(t) = H_0 x(t)$. Such decoupling is ensured by introducing dynamic feedback into the system, being implemented through a compensator generally described by the nonlinear equations

$$\begin{aligned} \dot{z}(t) &= \varphi(z(t), v(t), y(t)), \\ u(t) &= g_*(z(t), v(t), y(t)), \end{aligned} \tag{2.3}$$

where $z(t) \in \mathbb{R}^k$ denotes the state vector of a compensator of dimension $k < n$, $v(t)$ is the new control, and the functions φ and g_* have to be determined. Note that the variable $y_0(t)$ must be expressed through the state vector $z(t)$.

For the discrete-time analog of system (2.2), the problem of insensitivity to (or full decoupling from) disturbances and faults via feedback was solved in a general form in [18, 19] based on a rather complex mathematical apparatus of function algebra. In this paper, we solve the problem of full or partial decoupling with insensitivity (or minimal sensitivity) to faults for system (2.1) within the logical-dynamic approach [22], which operates only linear algebra methods.

The problem solution involves a submodel of system (2.1) insensitive or minimally sensitive to faults and a compensator built on its basis. Note that interval observers in [24] were designed by building a minimal-dimension submodel. In contrast, the compensator supplying the feedback is built based on a submodel of a maximal dimension $k < n$, which provides the best conditions for satisfying the equality $y_0(t) = H_0 x(t)$. This submodel is described by the equation

$$\dot{x}_*(t) = F_* x_*(t) + G_* u(t) + J_* y(t) + C_* \Psi_*(x_*(t), y(t), u(t)), \tag{2.4}$$

where $x_*(t) \in \mathbb{R}^k$ stands for the state vector of the submodel of dimension $k < n$; F_*, G_*, J_*, and C_* are the matrices to be determined;

$$C_* \Psi_*(x_*, y, u) = \begin{pmatrix} \varphi_{i_1}(A_{*1,i_1} x_* + A_{*2,i_1} y, u) \\ \dots \\ \varphi_{i_k}(A_{*1,i_k} x_* + A_{*2,i_k} y, u) \end{pmatrix}, \tag{2.5}$$

where A_{*1,i_1}, A_{*2,i_1}, …, A_{*1,i_k}, and A_{*2,i_k} are the matrices to be determined; $C_*\Psi_*$ denotes the function $C_*\Psi$ in which the vector x is replaced by x_* and y through the relation $A_i x = A_{*1,i} x_* + A_{*2,i} y$, where $i = i_1, \dots, i_k$ are the numbers of the nonzero columns of the matrix C_*.

3. BUILDING THE FAULT-INSENSITIVE SUBMODEL

We clarify that submodel (2.4) for building the compensator is a virtual object. In fact, it represents part of system (2.1) whose dynamics are determined by the state vector x_* related to the vector x by $x_*(t) = \Phi x(t)$, where Φ is some constant matrix. Generally speaking, these vectors can be related by a nonlinear function, and the assumption of its linearity restricts the class of solutions; it is characteristic of the logical-dynamic approach used here.

According to [22, 24], this matrix satisfies the equations

$$\begin{gathered} \Phi F = F_* \Phi + J_* H, \quad \Phi G = G_*, \quad \Phi C = C_*, \quad \Phi D = D_* \\ A_i = (A_{*1,i} \ \ A_{*2,i}) \begin{pmatrix} \Phi \\ H \end{pmatrix}, \quad i = i_1, \dots, i_k. \end{gathered} \tag{3.1}$$

The last equality in (3.1) is valid if

$$\operatorname{rank} \begin{pmatrix} \Phi \\ H \end{pmatrix} = \operatorname{rank} \begin{pmatrix} \Phi \\ H \\ A' \end{pmatrix}, \tag{3.2}$$

where the matrix A' consists of the rows A_{i_1}, …, A_{i_k}.

To solve the problem, we introduce the additional condition $y_0(t) = H_* x_*(t)$ for some matrix H_*, i.e., the variable $y_0(t) = H_0 x(t)$ must be expressed through the compensator state vector. In view of $x_*(t) = \Phi x(t)$, it follows that

$$\operatorname{rank} \begin{pmatrix} \Phi \end{pmatrix} = \operatorname{rank} \begin{pmatrix} \Phi \\ H_0 \end{pmatrix}. \tag{3.3}$$

If this condition fails, the problem is unsolvable. Under this condition, the matrix H_* is found from the equation $H_* \Phi = H_0$.

To ensure the fault-insensitivity condition $\Phi D = D_* = 0$, we introduce a matrix D_0 of maximal rank such that $D_0 D = 0$. Then $\Phi D = 0$ implies $\Phi = N D_0$ for some matrix N. Let us replace the matrix Φ in $\Phi F = F_*\Phi + J_*H$ with ND_0, i.e., $ND_0F = F_*ND_0 + J_*H$. After the separation of the unknown and known matrices, the resulting expression can be written as

$$\begin{pmatrix} N & -F_*N & -J_* \end{pmatrix} \begin{pmatrix} D_0F \\ D_0 \\ H \end{pmatrix} = 0. \tag{3.4}$$

Solving equation (3.4) yields the matrices F_*, J_*, and N, which are, in turn, allow finding the matrix Φ. Let the compound matrix $\begin{pmatrix} X & Y & Z \end{pmatrix}$ contain all linearly independent solutions of equation (3.4), i.e.,

$$\begin{pmatrix} X & Y & Z \end{pmatrix} \begin{pmatrix} D_0F \\ D_0 \\ H \end{pmatrix} = 0. \tag{3.5}$$

Comparing equations (3.4) and (3.5), we obtain the equality $Y = -F_*X$. Therefore, the matrices Y and X cannot be arbitrary: the rows of Y must be linearly expressed through the rows of X. To consider this fact, the rows of Y that are linearly independent of the rows of X must be removed. This procedure is implemented using Algorithm 1, where Y_j denotes the jth row of the matrix Y, $j = 1, \dots, p$, and p is the number of rows in the matrix Y.

Algorithm 1.

(1) Set $j = 1$.

(2) If $\operatorname{rank}(X) = \operatorname{rank}\begin{pmatrix} X \\ Y_j \end{pmatrix}$, pass to Step 4; otherwise, to Step 3.

(3) Remove the jth row from the matrix $\begin{pmatrix} X & Y & Z \end{pmatrix}$, set $p := p - 1$, and return to Step 1.

(4) If $j < p$, set $j := j + 1$ and return to Step 2; otherwise, complete the procedure.

Let $\begin{pmatrix} X_0 & Y_0 & Z_0 \end{pmatrix}$ denote the matrix outputted by the algorithm. For this matrix, the rows of the matrix Y_0 are linearly expressed through the rows of the matrix X_0. Letting $\Phi := X_0D_0$ and $C_* := \Phi C$, we construct the matrix A'; if the matrix Φ satisfies condition (3.2), a nonlinear fault-insensitive compensator can be built. Otherwise, full fault decoupling is unreachable, and robust methods should be used. If condition (3.3) fails for this matrix, the problem is unsolvable.

Letting $J_* = -Z_0$ and $G_* = \Phi G$, we find the matrix F_* from the algebraic equation $Y_0 = -F_*X_0$. It surely has a solution because, according to Algorithm 1, Y_0 is linearly expressed through the rows of the matrix X_0. Thus, the matrices describing the linear part of the submodel have been obtained. To construct the nonlinear part, we take $C_* = \Phi C$ and determine the matrices $A_{*1,i}$ and $A_{*2,i}$, $i = i_1, \dots, i_k$, from equation (3.1). This gives the nonlinear part (2.5) and, consequently, the entire submodel (2.4).

4. BUILDING THE ROBUST SUBMODEL

If $\begin{pmatrix} X_0 & Y_0 & Z_0 \end{pmatrix} = 0$ or the matrix Φ does not satisfy condition (3.2), the fault-insensitive compensator cannot be built. In this case, it is necessary to address robust methods to minimize the fault contribution to model (2.4). For this purpose, we write the relation $\Phi F = F_*\Phi + J_*H$ in a form similar to (3.3), removing the fault-insensitivity constraint $\Phi D = D_* = 0$ and separating the unknown matrices from the known ones:

$$\begin{pmatrix} \Phi & -F_*\Phi & -J_* \end{pmatrix} \begin{pmatrix} F \\ E \\ H \end{pmatrix} = 0, \tag{4.1}$$

where E is an identity matrix of appropriate dimensions. Now equation (4.1) can have solutions admitting the model's sensitivity to faults.

As above, we consider the compound matrix $\begin{pmatrix} X & Y & Z \end{pmatrix}$ containing all linearly independent solutions of equation (4.1), i.e.,

$$\begin{pmatrix} X & Y & Z \end{pmatrix}\begin{pmatrix} F \\ E \\ H \end{pmatrix} = 0.$$

Applying Algorithm 1 to the matrix $\begin{pmatrix} X & Y & Z \end{pmatrix}$, we obtain the matrix $\begin{pmatrix} X_* & Y_* & Z_* \end{pmatrix}$ in which $Y_* = -MX_*$ with some matrix M. If this equation has several solutions, they will correspond to several matrices Φ : $\Phi^{(1)}, \ldots, \Phi^{(s)}$. By determining, for each of them, the norm $\|\Phi^{(i)}D\|$ corresponding to the fault contribution to the compensator, we can choose the variant with the smallest norm value corresponding to the minimal fault contribution to the submodel.

A better result can be obtained by setting the matrix $\Phi = \sum_{i=1}^{s} v_i\Phi^{(i)}$ and assigning the weights $v_1, \ldots, v_s$ based on minimization of the norm $\|\Phi D\|$. However, this approach is possible only if the matrix F_* in the expression $\Phi F = F_*\Phi + J_*H$ remains the same for different Φ. We implement this approach by choosing F_* in the canonical form

$$F_* = \begin{pmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \\ 0 & 0 & 0 & \ldots & 0 \end{pmatrix}, \tag{4.2}$$

which will additionally simplify the design procedure. Due to the canonical form (4.2), equations (3.1) become [22]

$$\Phi_i F = \Phi_{i+1} + J_{*i}H, \quad i = 1, \ldots, k-1, \quad \Phi_k F = J_{*k}H, \tag{4.3}$$

where Φ_i and J_{*i} are the ith rows of the matrices Φ and J_*, respectively, $i = 1, \ldots, k$. According to [22], these equations can be convolved into one:

$$\begin{pmatrix} \Phi_1 & -J_{*1} & -J_{*2} & \ldots & -J_{*k} \end{pmatrix} V^{(k)} = 0, \tag{4.4}$$

where

$$V^{(k)} = \begin{pmatrix} HF^k \\ HF^{k-1} \\ \ldots \\ H \end{pmatrix}.$$

Also, see [22], the minimization problem of the fault contribution to the submodel reduces to minimizing the norm $\|\Phi D\| = \|\begin{pmatrix} \Phi_1 & -J_{*1} & -J_{*2} & \ldots & -J_{*k} \end{pmatrix} D^{(k)}\|$ subject to condition (4.4), where

$$D^{(k)} = \begin{pmatrix} D & FD & F^2D & \ldots & F^{k-1}D \\ 0 & HD & HFD & \ldots & HF^{k-2}D \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 0 \end{pmatrix}.$$

When solving this problem, we find a maximal dimension $k < n$ for which equation (4.4) has several (more than one) linearly independent solutions of the form $\begin{pmatrix} \Phi_1 & -J_{*1} & -J_{*2} & \ldots & -J_{*k} \end{pmatrix}$.

All these solutions, s totally, are combined into a matrix W so that each row represents some solution of equation (4.4):

$$W = \begin{pmatrix} \Phi_1^{(1)} & -J_{*1}^{(1)} & -J_{*2}^{(1)} & \dots & -J_{*k}^{(1)} \\ \dots & \dots & \dots & \dots & \dots \\ \Phi_1^{(s)} & -J_{*1}^{(s)} & -J_{*2}^{(s)} & \dots & -J_{*k}^{(s)} \end{pmatrix}.$$

Due to the considerations above, another solution is an arbitrary linear combination of the rows of this matrix with the vector of weights $v = (v_1, \dots, v_s)$. The problem is to determine such a vector v that minimizes the norm $\|vWD^{(k)}\|$.

To solve this problem, we find the singular value decomposition of the matrix product $WD^{(k)}$:

$$WD^{(k)} = U_D \Sigma_D V_D,$$

where U_D and V_D are orthogonal matrices; depending on the numbers of rows and columns in the matrix $WD^{(k)}$, the matrix Σ_D has the form

$$\Sigma_D = (\mathrm{diag}(\sigma_1, \dots, \sigma_w)\ 0)$$

or

$$\Sigma_D = \begin{pmatrix} \mathrm{diag}(\sigma_1, \dots, \sigma_w) \\ 0 \end{pmatrix},$$

with $w = \min(s, k)$ and $0 \leqslant \sigma_1 \leqslant \dots \leqslant \sigma_w$ being the singular values of the matrix $WD^{(k)}$ [22, 25]. The first transposed column of the matrix U_D is chosen as the vector of weights $v = (v_1, \dots, v_s)$. By the structure of singular value decomposition and the properties of orthogonal matrices, the norm of the matrix $vWD^{(k)}$ equals the minimal singular value σ_1 [22], and $(\ \Phi_1\ \ -J_{*1}\ \ -J_{*2}\ \ \dots\ \ -J_{*k}\ ) = vW$. Then the rows of the matrix Φ are determined from (4.3) and the matrix A' is constructed. If this matrix satisfies conditions (3.2) and (3.3), we take $G_* = \Phi G$ and $C_* = \Phi C$ and find the matrices $A_{*1,i}$ and $A_{*2,i}$, $i = i_1, \dots, i_k$, from equation (3.1); this completes the robust model design. Note that this solution will be optimal for the chosen dimension k; changing the dimension may yield a better solution of the problem in terms of minimizing the norm $\|(\Phi_1 - J_{*1} - J_{*2}\ \dots - J_{*k})D^{(k)}\|$. If condition (3.2) or (3.3) fails, it is necessary to choose the second or subsequent transposed columns of the matrix U_D.

5. BUILDING THE COMPENSATOR

To avoid confusion, we denote the compensator state vector by $z(t) := x_*(t)$, leaving unchanged the notations for the other elements, particularly the matrix H_* and the function f_*.

From this point onwards, condition (3.3) is assumed valid, i.e., $y_0 = H_* z$. We denote by X_y the set of components of the vector z participating in the formation of y_0. For building the compensator, model (2.4) will be written in a compact form:

$$\dot{z}(t) = f_*(z(t), u(t), y(t)). \tag{5.1}$$

Even if this model does not explicitly contain the unknown function $d(t)$ (when full decoupling is reached), its state vector is affected by faults due to the presence of the vector $y(t)$ in (5.1). To build the compensator, this effect must be eliminated by adjusting the control vector $u(t)$ via feedback with a new control vector $v(t)$. The algorithm below performs the necessary analysis and generates the feedback if possible. Let f_{*j} denote the jth component of the function f_*.

Algorithm 2.

(1) Divide the components of the vector y into two disjoint sets, Y_g (*good*) and Y_b (*bad*), according to the rules: the variable y_i is included in Y_g if it does not appear in the function f_* or can be expressed through the components of the vector z; otherwise, y_i is included in Y_b. If $Y_b = \emptyset$, full or partial fault decoupling is reached without the compensator since y_i in the function f_* can be replaced by a function of the vector z.

(2) If $Y_b \neq \emptyset$, for each $y_i \in Y_b$ find a variable z_j such that f_{*j} depends on y_i and is independent of u. Let X_b denote the set of all such z_j; it consists of all components of the state vector that are affected by the fault because f_{*j} includes the variable y_i not compensated by the control. If $X_b = \emptyset$, pass to Step 4.

(3) For each $z_j \in X_b$ find the functions f_{*i} that depend on z_j. If all f_{*i} depend on u, add z_j to Y_b and remove it from X_b. If for some i this condition fails, then the variable z_i cannot be decoupled from faults; if $z_i \in X_y$, i.e., this variable participates in the formation of the variable y_0, then the problem has no solution. If $z_i \notin X_y$, add z_i to X_b and continue executing Step 3 until $X_b = \emptyset$ or X_b stops changing. The final set Y_b contains the variables that will participate in the feedback to compensate for the effect of faults.

(4) Find in the function $f_*(z, u, y)$ all terms of the form $\gamma_i(z, u, y)$, $i = 1, \ldots, r$, that depend on u and elements from the set Y_b; by assumption, $r \leqslant m$. Form a system of equations for the new control vector $v = (v_1 \ \ldots \ v_m)^T$:

$$\begin{aligned} v_1 &= \gamma_1(z, u, y), \\ &\cdots \\ v_r &= \gamma_r(z, u, y). \end{aligned}$$

Supposing the feasibility of this system with respect to the variables $u_1, \ldots, u_r$, find its solution:

$$\begin{aligned} u_1 &= \gamma_1(z, u, y, v), \\ &\cdots \\ u_r &= \gamma_r(z, u, y, v); \\ u_{r+1} &= v_{r+1}, \ \ldots, \ u_m = v_m. \end{aligned} \tag{5.2}$$

Replace the vector u in (5.1) with the vector v according to the rules (5.2), which gives the dynamic part of the compensator (2.3); its static part coincides with (5.2).

6. EXAMPLE

Consider the nonlinear system

$$\begin{aligned} \dot{x}_1 &= u_1/\vartheta_1 - a_1\sqrt{x_1 - x_2} - d, \\ \dot{x}_2 &= u_2/\vartheta_2 + a_1\sqrt{x_1 - x_2} - a_2\sqrt{x_2 - x_3}, \\ \dot{x}_3 &= a_2\sqrt{x_2 - x_3} - a_3\sqrt{x_3 - \vartheta_7}, \\ y &= x_1, \end{aligned} \tag{6.1}$$

where $a_1 = \vartheta_4\sqrt{2\vartheta_8}/\vartheta_1$, $a_2 = \vartheta_5\sqrt{2\vartheta_8}/\vartheta_2$, and $a_3 = \vartheta_6\sqrt{2\vartheta_8}/\vartheta_3$. These equations describe the known three-tank system (Fig. 2), where x_1, x_2, and x_3 are the liquid levels in the tanks [26]. The system consists of three tanks with cross sections ϑ_1, ϑ_2, and ϑ_3, respectively; the tanks are interconnected by pipes with cross sections ϑ_4 and ϑ_5. The liquid flows in the first and second tanks, flowing out of the third one through a pipe of a cross section ϑ_6 located at a height ϑ_7; the parameter ϑ_8 is the gravitational constant. The controls u_1 and u_2 correspond to the externally supplied fluid. A nonzero value $d(t) > 0$ corresponds to leakage in the first tank; the variable

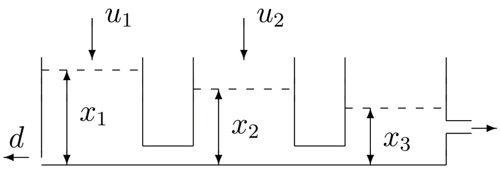

Fig. 2. A three-tank system.

$y_0(t) = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} x(t) = x_3(t)$ must be insensitive to it. The amount of leakage is assumed to be unknown, so it cannot be compensated for by increasing u_1 and the proposed method should be used instead.

For the sake of simplicity, let $a_1 = a_2 = a_3 = 1$ and $\vartheta_7 = 0$. The initial conditions and control are supposed to be such that $x_1(t) \geqslant x_2(t) \geqslant x_3(t) \geqslant 0$ for all $t \geqslant 0$.

Clearly, $F = 0$ for (6.1), and the considered approach cannot be applied directly. To overcome this difficulty, we transform (6.1) by introducing the formal terms $-(x_1 - x_2) + (x_1 - x_2)$, $((x_1 - x_2) - (x_2 - x_3)) - ((x_1 - x_2) - (x_2 - x_3))$, and $(x_2 - x_3 - x_3) - (x_2 - x_3 - x_3)$ into the first, second, and third equations, respectively. The term $-(x_1 - x_2)$ is added to the linear part; the term $(x_1 - x_2)$, to the nonlinear part. The remaining terms are handled similarly. As a result, the system is described by the following matrices and nonlinearities:

$$F = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix}, \; G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \; H = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, \; H_0 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix},$$

$$D = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}, \; C = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}, \; \Psi(x) = \begin{pmatrix} -\sqrt{A_1 x} + A_1 x \\ -\sqrt{A_2 x} + A_2 x \\ -\sqrt{A_3 x} + A_3 x \end{pmatrix},$$

$$A_1 = (1 \;\; -1 \;\; 0), \;\; A_2 = (0 \;\; 1 \;\; -1), \;\; A_3 = (0 \;\; 0 \;\; 1).$$

Since $D = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^T$, we have $D_0 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, and equation (3.5) takes the form

$$\begin{pmatrix} X & Y & Z \end{pmatrix} \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = 0.$$

The solution is

$$\begin{pmatrix} X & Y & Z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & -1 & 2 & 0 \end{pmatrix}.$$

As is easily verified, the condition of Step 2 of Algorithm 1 holds for both rows of the matrix Y. Therefore,

$$\begin{pmatrix} X_0 & Y_0 & Z_0 \end{pmatrix} = \begin{pmatrix} X & Y & Z \end{pmatrix},$$

and consequently,

$$J_* = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad F_* = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}, \quad \Phi = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad G_* = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

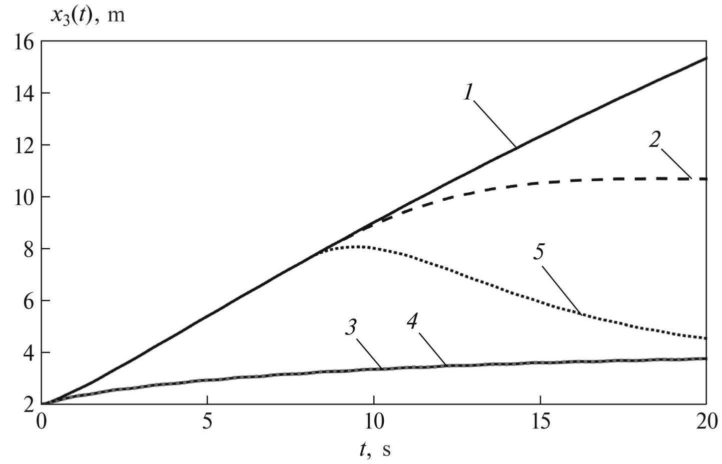

Fig. 3. The behavior of the variable $x_3(t) = y_0(t)$.

In view of $H_0 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$, condition (3.3) is obviously valid; the matrix H_* is found from the equation $H_0 = H_*\Phi$ and has the form $H_* = (0\ \ 1)$.

As a result, the linear part of submodel (2.4) is described by the equations

$$\dot{x}_{*1} = u_2 - 2x_{*1} + x_{*2} + y,$$
$$\dot{x}_{*2} = x_{*1} - 2x_{*2},$$

where $x_{*1} = \Phi_1 x = x_2$ and $x_{*2} = \Phi_2 x = x_3$. In addition, $y_0 = H_* x_* = x_{*2}$, i.e., $X_y = \{x_{*2}\}$.

All columns in the matrix $C_* = \Phi C = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$ are nonzero, and the matrix A' hence contains three rows A_1, A_2, and A_3; condition (3.2) holds for it. Solving equation (3.1) yields

$$A_{*1,1} = (-1\ \ 0),\ A_{*2,1} = 1,\ A_{*1,2} = (1\ \ -1),\ A_{*2,2} = 0,\ A_{*1,3} = (0\ \ 1),\ A_{*2,1} = 0.$$

Therefore, the nonlinear part (2.5) takes the form

$$C_*\Psi_*(x_*, y, u) = \begin{pmatrix} \sqrt{y - x_{*1}} - (y - x_{*1}) - \sqrt{x_{*1} - x_{*2}} + (x_{*1} - x_{*2}), \\ \sqrt{x_{*1} - x_{*2}} - (x_{*1} - x_{*2}) - \sqrt{x_{*2}} + x_{*2} \end{pmatrix}.$$

Finally, adding it to the linear part gives the nonlinear submodel

$$\begin{aligned} \dot{x}_{*1} &= u_2 + \sqrt{y - x_{*1}} - \sqrt{x_{*1} - x_{*2}}, \\ \dot{x}_{*2} &= \sqrt{x_{*1} - x_{*2}} - \sqrt{x_{*2}}. \end{aligned} \tag{6.2}$$

Since $y = x_1$ is not expressed through the vector $z := x_*$, Step 1 of Algorithm 2 yields $Y_g = \emptyset$ and $Y_b = \{y\}$. Step 2 of this algorithm leads to $X_b = \emptyset$; Step 4 yields $r = 1$ and the single equation $v_2 = u_2 + \sqrt{y - z_1}$, which is obviously solvable for u_2 :

$$u_2 = v_2 - \sqrt{y - z_1}.$$

Setting $v_1 = u_1$ and substituting the above formula for u_2 into (6.2), we finally arrive at the compensator description

$$\begin{aligned} \dot{z}_1 &= v_2 - \sqrt{z_1 - z_2}, \\ \dot{z}_2 &= \sqrt{z_1 - z_2} - \sqrt{z_2}, \\ u_1 &= v_1, \\ u_2 &= v_2 - \sqrt{y - z_1}. \end{aligned} \tag{6.3}$$

For numerical simulation, we select $u_1(t) = 5$ and $u_2(t) = 2\sin(5t)$. Figure 3 shows the behavior of the variable $x_3(t) = y_0(t)$ of system (6.1) with the initial state $x(0) = 0$ for five different cases. Curve *1* corresponds to the case without the fault and decoupling; curve *2*, to the case where the fault $d = 4$ occurs at the time instant $t = 8$, but fault decoupling is not introduced (the variable changes its dynamics for $t > 8$). Curves *3* and *4* correspond to the introduction of decoupling with $v_2(t) = 2 + \sin(5t)$ at the time instant $t = 0$ in the system without the fault and with the fault, respectively; since curves *3* and *4* coincide, the fault is not manifested (has no effect on $x_3(t)$). Curve *5* corresponds to the system with the fault and decoupling with $v_2(t) = 2 + \sin(5t)$ introduced at the time instant $t = 8$; until this instant the behavior of the variable $y_0(t)$ coincides with curve *1*.

Clearly, curves *3* and *4*, where the decoupling with $v_2(t) = 2 + \sin(5t)$ is introduced at the time instant $t = 0$, do not coincide with curve *1* (the behavior of the variable without the fault). To achieve this coincidence, it is necessary to solve the control problem for the variable $v_2(t)$ in system (6.1) with the compensator (6.3). This is an independent problem solved by known methods. A similar picture is observed in case 5: when the fault occurs and the compensator is introduced, the variable $y_0(t)$, $t > 8$, changes its behavior, and the coincidence with its dynamics without the fault can be achieved by solving the control problem for the variable $v_2(t)$.

7. CONCLUSIONS

This paper has considered technical systems described by nonlinear dynamic models. The fault tolerance property of such systems has been ensured by introducing feedback with full or partial fault decoupling. The solution is based on the logical-dynamic approach, which operates only linear algebra methods. An illustrative practical example has been provided.

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